APPROXIMATE SYMMETRIES IN NUCLEI AND THE $2\nu\beta\beta$ - DECAY RATE

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A nonstandard method for calculating the nuclear $2\nu\beta\beta$ - decay amplitude is proposed. The method is based on the explicit use of those approximate symmetries of a nuclear hamiltonian, which correspond to the operators of allowed $\beta-$ transitions. Within the framework of the proposed method the mentioned amplitude is calculated for a wide range of nuclei. The model parameters used in calculations are taken from independent data. Calculated $2\nu\beta\beta$ half-lifes are compared with known experimental data.

1. An analysis of a great body of known experimental data (see e.g. ref. [1]) allows one to conclude that the $2\nu\beta\beta$ - decay rate is hindered as compared with the rates evaluated within the independent-quasiparticle approximation. The hindrance is caused by existence of the isobaric analogue and Gamow-Teller nuclear collective states (IAS and GTS) with relatively large excitation energy. These states each exhaust the main part of the Fermi and Gamow-Teller strength, respectively. To describe the hindrance the different versions of the quasiparticle random phase approximation (QRPA) are used for calculations of the nuclear $2\nu\beta\beta$ - decay amplitude. It has been found (see e.g. refs. [2, 3, 4]) that the calculated amplitudes are very sensitive to the particle-particle interaction strength. For this reason the predictable power of the QRPA current versions is poor. As reported recently, the similar instability of the calculated $0\nu\beta\beta$ - decay amplitude has been also found [5, 6]. We believe that the mentioned difficulty caused by inconsistency of the QRPA current versions can be presently overcome by the explicit use of the approximate isospin and spin-isospin symmetries of a nuclear hamiltonian. Early attempts along this line have been undertaken in refs. [7, 8].

It is well-known that the Fermi-type excitations can be correctly described only with consideration for the isospin symmetry of a nuclear hamiltonian. Such a consideration allows one to reproduce in model calculations the experimental fact, that IAS exhausts more than 95% the Fermi sum rule $N-Z$, as well as to evaluate correctly the small Fermi-strength of the low-energy IAS satellites populated virtually in the $2\nu\beta\beta$ - decay process. If the isospin symmetry were exact, the IAS would exhaust 100% of the Fermi strength and the Fermi nuclear amplitude for the $2\nu\beta\beta$ - decay to the ground or low-excited states of a product nucleus would be equal to zero. The above statements allow one to describe the Fermi-type excitations within the framework of the perturbation theory with respect to the variable part of the nuclear mean Coulomb field. This latter is the main source of the isospin-symmetry violation in intermediate and heavy mass nuclei (see e.g. ref. [9] and refs. therein).

The similar situation takes place at the description Gamow-Teller (GT) excitations. It is experimentally established that in intermediate and heavy mass nuclei the GTS exhausts about 70% - 80% sum rule $3e_q^2(N-Z)$, where $e_q \simeq 0.8$ is "a effective charge" describing, in particular, the renormalization of the axial constant of the weak interaction in nuclei. The mentioned experimental fact can be considered as an evidence of approximate conservation of the appropriate spin-isospin symmetry of a nuclear hamiltonian (see in this connection ref. [10]). This symmetry should be taken into consideration for correct description of both the GTS low-energy satellites and the nuclear $2\nu\beta\beta$ - decay amplitude. If the mentioned symmetry were exact, the amplitude would be equal to zero. Because the $2\nu\beta\beta$ -decay hindrance is connected with formation of the collective GTS, the hindrance can be also considered as a result of partial conservation of the appropriate spin-isospin symmetry.

In the presented work a method for calculating the GT nuclear $2\nu\beta\beta$ - decay amplitude is proposed. The method is based on the explicit consideration for approximate conservation of the symmetries corresponding to operators of the allowed Fermi and GT β - transitions. The $2\nu\beta^-\beta^-$ and $2\nu\beta^+\beta^+$ (including the electron capture) half-lifes for a wide range of nuclei are calculated within the framework of the simplest version of the proposed method. The model parameters used in calculation are taken from independent data. Calculation results are compared with known experimental data.

2. To clarify the main point of the proposed method we derive the basic formula for nuclear $2\nu\beta\beta$ - decay amplitude by the same way for both the Fermi and GT amplitudes. At first we consider the $2\nu\beta^-\beta^-$ - decay. Let $|i\rangle$ and $|f\rangle$ be the wave functions of the ground state, respectively, of a double-even parent-nucleus $(N+2, Z)$ with energy E_i and of a product-nucleus $(N, Z+2)$ with the energy E_f, $G^{(\pm)}$ be the operators of allowed Fermi ($G^{(\pm)} = \sum_a \tau^{(\pm)}(a)$) or Gamow-Teller ($G^{(\pm)} = \sum_a \vec{\sigma}(a)\tau^{(\pm)}(a)$) β - transitions.

We start with the zero approximation, in which symmetries corresponding to operators $G^{(\pm)}$ are exact. In this approximation the states $|G\rangle$ (IAS and GTS) of an isobaric nucleus $(N+1, Z+1)$: (i) each exhaust 100% of the corresponding sum rule, so that

$$|G\rangle = k_G\, G^{(-)}|i\rangle, \quad G^{(+)}|i\rangle = G^{(+)}|f\rangle = 0;\; k_G = \langle i|[G^{(+)}, G(-)]|i\rangle^{-1/2}\ ; \tag{1}$$

(ii) have the same energy E_G; (iii) are orthogonal to other states $|A\rangle$ of the same isobaric nucleus:

$$\langle G|A\rangle = 0; \quad G^{(+)}|A\rangle = 0\ . \tag{2}$$

For the further analysis of the nuclear $2\nu\beta\beta$ - decay amplitudes it is necessary to consider also the following excited states of a product-nucleus:

$$|GA\rangle = k_A\, G^{(-)}|A\rangle, \quad E_{GA} = E_G - E_i + E_A, \quad k_A = \langle A|[G^{(+)}, G^{(-)}]|A\rangle^{-1/2}\ , \tag{3}$$

$$|DG\rangle = k_D\ G^{(-)}G^{(-)}|i\rangle, \quad E_{DG} = E_G - E_i + E_G, \quad k_D = \langle i|G^{(+)}G^{(+)}G^{(-)}G^{(-)}|i\rangle^{-1/2}\ . \tag{4}$$

Low-energy states $|A\rangle$ are called the anti-IAS or anti-GTS, whereas states $|DG\rangle$ are called the double-IAS or double-GTS.

Let U be the nuclear mean field, so that $U = \sum_a U(\mathbf{r}_a)$ and $[U, G^{(-)}] = U_G^{(-)} \neq 0$. The part of the nuclear mean field $U_G = \sum_a U_G(\mathbf{r}_a)$, which leads to nonzero operator $U_G^{(-)}$, is the main source of violating the symmetry connected with operator $G^{(-)}$. Other sources are less essential and briefly considered below. In the lowest order in the U_G the energy of state (1) is equal to

$$E_G - E_i = k_G^2\langle i|[G^{(+)}, U_G^{(-)}]|i\rangle \equiv \Delta_G\ . \tag{5}$$

Let us turn to the expression for the nuclear $2\nu\beta^-\beta^-$ - decay amplitude (see e.g. refs. [11, 12]):

$$M_G = \sum_S \langle f|G^{(-)}|S\rangle\langle S|G^{(-)}|i\rangle\omega_S^{-1}\ , \tag{6}$$

where the sum is taken over intermediate states of an isobaric nucleus $(N+1, Z+1)$; $\omega_S = E_S - (E_i + E_f)/2$ is the excitation energy of the intermediate state. The $2\nu\beta^+\beta^+$ - decay amplitude is equal to conjugate value M_G^*. In the zero approximation the $2\nu\beta\beta$ - decay is doubly forbidden ($M_G = 0$), as it follows from eqs. (1)-(6) after replacing $|S\rangle \to |A\rangle, |G\rangle$. Mixing of nuclear states due to the single-particle field U_G results in nonzero value of amplitude M_G. We keep those corrections to wave functions of isobaric nuclei, which provide the contribution to the amplitude M_G of the same order perturbation with respect to U_G:

$$|G\rangle \to |G\rangle + \sum_A \alpha_{A,i}|A\rangle; \quad |A\rangle \to |A\rangle + \alpha_{i,A}|G\rangle\ ; \quad \alpha_{A,i} = -\alpha^*_{i,A};$$
$$|f\rangle \to |f\rangle + \sum_A \alpha_{A,f}|GA\rangle + \beta_{i,f}|DG\rangle\ . \tag{7}$$

With the help of eqs. (1)-(4) the first-order mixing amplitudes α and the second-order amplitude β can be expressed in terms of matrix elements of the single-particle charge-exchange field $U_G^{(-)}$:

$$\alpha_{A,i} = \frac{\langle A|U|G\rangle}{E_G - E_i} = k_G\frac{\langle A|U_G^{(-)}|i\rangle}{\omega_G - \omega_A}, \quad \alpha^*_{A,f} = \frac{\langle f|U|GA\rangle}{E_f - E_{GA}} = -k_A\frac{\langle f|U_G^{(-)}|A\rangle}{\omega_G + \omega_A},$$
$$\beta^*_{i,f} = \sum_A \frac{\langle f|U|GA\rangle\langle GA|U|DG\rangle}{(E_{GA} - E_f)(E_{DG} - E_f)} = k_D\sum_A \frac{\langle f|U_G^{(-)}|A\rangle\langle A|U_G^{(-)}|i\rangle}{(\omega_G + \omega_A)\omega_G}\ . \tag{8}$$

After substitution of the eqs. (7),(8) in eq. (6) with the use of eqs. (1)–(4) we get:

$$M_G = \omega_G^{-2} \sum_A \langle f|U_G^{(-)}|A\rangle\langle A|U_G^{(-)}|i\rangle\omega_A^{-1} \ .$$

This equation can be presented in the form, which is similar to the starting expression (6):

$$M_G = \omega_G^{-2} \sum_S \langle f|V_G^{(-)}|S\rangle\langle S|V_G^{(-)}|i\rangle\omega_S^{-1} \ , \tag{9}$$

where $V_G^{(-)} \equiv U_G^{(-)} - \Delta_G G^{(-)}$, Δ_G is determined by eq. (5). Indeed, the equality $\langle G|V_G^{(-)}|i\rangle = 0$ and eqs. (1),(2) allow one to include formally the state $|G\rangle$ in sum (9).

The fundamental character of formula (9) is explained by the following. (i) In this formula the approximate symmetries of a nuclear hamiltonian are explicitly taken into account. (ii) The hindrance of the strength of intermediate β – transitions due to formation of the collective states (IAS and GTS) is correctly taken into consideration in rather general form. (iii) Inasmuch as the main part of polarization effects caused by a quasiparticle interaction has been taken into account, rather simple nuclear models can be used for calculating the intermediate-state wave functions and energies in eq. (9).

3. As the first step in evaluation of the $2\nu\beta\beta$ - decay amplitude according to eq. (9) we use: (i) realistic phenomenological mean nuclear field with consideration for partial self-consistency conditions; (ii) the BCS model for nuclei with strong nucleon pairing; (iii) the pair-vibration model for description of the "magic $\pm$ two nucleon" subsystem.

We use the following phenomenological quantities involved in the model hamiltonian. The isoscalar part of the nuclear mean field $U_0(\mathbf{r})$ is the sum of central and spin-orbital parts:

$$U_0(\mathbf{r}) = U_0(r) + U_{SO}(\mathbf{r}); \quad U_{SO}(\mathbf{r}) = U_{SO}(r)(\mathbf{ls}) \ . \tag{10}$$

As an isovector part of the particle-hole interaction the Landau-Migdal forces are used:

$$\mathcal{F}(\mathbf{r}_1, \mathbf{r}_2) = (F' + G'\vec{\sigma}_1\vec{\sigma}_2)\vec{\tau}_1\vec{\tau}_2\delta(\mathbf{r}_1 - \mathbf{r}_2) \ , \tag{11}$$

where F' and G' are phenomenological constants. Other parts of the nuclear mean field are calculated by the self-consistent way. As a result of the isospin symmetry of the nuclear hamiltonian, the self-consistency condition, which relates the isovector part of nuclear mean field $U_{SY} = \frac{1}{2}\tau^{(3)}v(r)$ and neutron-excess density $\rho(r) = \rho^n(r) - \rho^p(r)$ via the interaction (11), takes place (see e.g. ref. [13]): $v(r) = 2F'\rho(r)$. The nuclear mean Coulomb field $U_C(r)$ is calculated within the Hartree approximation via the proton density ρ^p.

As mentioned above, the mean Coulomb field $U_C = \sum_a U_C(r_a)(1 - \tau^{(3)}(a))/2$ is the main source of the isospin-symmetry violation in intermediate and heavy mass nuclei. In particular, this field

results in the shift of the IAS energy from energy E_i. According to eq. (5) the shift equals the Coulomb displacement energy Δ_C:

$$E_{IAS} - E_i = \Delta_C = (N+2-Z)^{-1} \int U_C(r)\rho(r)d\mathbf{r}. \tag{12}$$

Here $\rho(r)$ is the above-mentioned neutron-excess density in the ground state of the parent nucleus:

$$\rho(r) = \frac{1}{4\pi}\left(\sum_\nu R_\nu^2(r)(2j_\nu+1)n_\nu - \sum_\pi R_\pi^2(r)(2j_\pi+1)n_\pi\right), \tag{13}$$

where $R_\lambda(r)$ are the radial single-neutron ($\lambda=\nu$) and single-proton ($\lambda=\pi$) wave functions; $\lambda = n_r, l, j$ is the set of the single-particle quantum numbers; n_λ are occupation numbers satisfying the equations:

$$\sum_\nu (2j_\nu+1)n_\nu = N+2, \quad \sum_\pi (2j_\pi+1)n_\pi = Z. \tag{14}$$

For nuclei with strong pairing in any nucleon subsystem the occupation numbers are $n_\lambda = v_\lambda^2 = 1-u_\lambda^2$, where v_λ and u_λ are the Bogoliubov-transformation coefficients. In the case that a nucleon subsystem is "magic $\pm$ two nucleons" one the occupation factors are: $n_\lambda = n_\lambda^m + c_\lambda^2(1-2n_\lambda^m)$, where n_λ^m are the occupations numbers for the magic subsystem; c_λ are the coefficients determining the pair-vibration wave function and satisfying the normalization condition: $\sum_\lambda c_\lambda^2(1-2n_\lambda^m)(2j_\lambda+1) = \pm 2$.

In view of a high degree of the isospin-symmetry conservation in nuclei we omit the analysis of the relatively small Fermi- $2\nu\beta\beta$ – decay amplitude and turn to the GT amplitude M_{GT}. The mean Coulomb field results in the same (in the lowest order of the perturbation theory) energy shift Δ_C (12) for the both IAS and GTS. However, the main source of violation of the spin-isospin symmetry corresponding to operator $G^{(-)} = \sum_a \vec{\sigma}_a \tau_a^{(-)}$ is the spin-orbital part of nuclear mean field (10): $U_{SO} = \sum_a U_{SO}(\mathbf{r}_a)$. In particular, this field results in the energy shift Δ_{SO} of the GTS from the IAS energy. According to eq. (5) we have:

$$\Delta_{SO} = \frac{1}{3(N+2-Z)}\langle i|[G^{(+)}[U_{SO}, G^{(-)}]]|i\rangle = -\frac{4}{3(N+2-Z)}\langle i|U_{SO}|i\rangle;$$
$$\langle i|U_{SO}|i\rangle = \sum_{\lambda=\pi,\nu} \langle\lambda|U_{SO}(r)|\lambda\rangle(\mathbf{ls})_{j_\lambda l_\lambda}(2j_\lambda+1)n_\lambda\ . \tag{15}$$

Here $\langle\pi|U_{SO}(r)|\nu\rangle$ is the radial matrix element; n_λ are occupation numbers satisfying eqs. (14).

In the case of GT excitations the mixing (spin-orbital) field in eq. (9) has the form: $V_G^{(-)} = \sum_a V_{SO}(a)\tau_a^{(-)}$, where $V_{SO}(a) = U_{SO}(r_a)[\mathbf{ls}, \vec{\sigma}] - \Delta_{SO}\vec{\sigma}$. Calculation of amplitude M_{GT} according to eq. (9) within the framework of the BCS model results in the expression:

$$M_{GT} = e_q^2 \omega_{GTS}^{-2} \sum_{\pi,\nu} \left((2l_\pi + 1)(j_\pi - j_\nu) \langle \pi | U_{SO}(r) | \nu \rangle - \Delta_{SO} \langle \pi | \nu \rangle \right)^2 \langle \pi \| \sigma \| \nu \rangle^2 u_\pi v_\pi u_\nu v_\nu \omega_{\pi\nu}^{-1} \ . \qquad (16)$$

Here $\langle \pi | \nu \rangle = \int R_\pi(r) R_\nu(r) r^2 dr$ is the overlap integral; $\langle \pi \| \sigma \| \nu \rangle$ is the reduced matrix element; $\omega_{\pi\nu} = \mathcal{E}_\pi + \mathcal{E}_\nu$ is the excitation energy of the two-quasiparticle state, $\mathcal{E}_\lambda = \sqrt{(\epsilon_\lambda - \mu)^2 + \Delta^2}$ is the single-quasiparticle energy for subsystem with strong nucleon pairing, ϵ_λ is the energy of single-particle level, Δ is the energy gap. If nucleon subsystem is magic in final (initial) state and "magic $\pm$ two nucleon" in initial (final) state the following replacements in eq. (16) should be made: $u_\lambda v_\lambda \rightarrow c_\lambda$ and $\mathcal{E}_\lambda = |\Delta + \epsilon_\lambda - \epsilon_1|$ for particle pair-vibrations or $\mathcal{E}_\lambda = |\Delta + \epsilon_0 - \epsilon_\lambda|$ for hole pair-vibrations, where ϵ_1 (ϵ_0) is the energy of the first empty (last filled) single-particle level in the magic subsystem; 2Δ is the pair-vibration state energy, which coincides with the pair energy P for the subsystem "magic $\pm$ two nucleons". The hindrance factor h_{GT}, which can be considered as a perturbation theory parameter, is estimated as follows: $h_{GT} = M_{GT}/M_{GT}^0$, where amplitude M_{GT}^0 is evaluated accordind to eq. (6) within the framework of the BCS model without consideration for the spin-isospin symmetry:

$$M_{GT}^0 = e_q^2 \sum_{\pi,\nu} \langle \pi | \nu \rangle^2 \langle \pi \| \sigma \| \nu \rangle^2 u_\pi v_\nu u_\nu v_\nu \omega_{\pi\nu}^{-1} \ . \qquad (17)$$

In the conclusion of this section we briefly consider other sources of the spin-isospin symmetry violation of the nuclear hamiltonian. The inequality $F' \neq G'$ in eq. (11) results in appearance of the mixing field $V_\delta^{(-)} = \delta \sum_a v(r_a) \tau^{(-)}(a)$, where $\delta = (G' - F')/F'$, $v(r)$ is the symmetry potential considered previously. In view of smooth radial dependence of $v(r)$ (as well as in the case of mean Coulomb field U_C) the contribution of field $V_\delta^{(-)}$ to the GTS energy should be only taken into account [10]. By analogy to eq. (12) we have:

$$\Delta_\delta = \delta (N + 2 - Z)^{-1} \int v(r) \rho(r) d\mathbf{r} \ .$$

Thus, the GTS excitation energy in eq. (16) equals $\omega_{GTS} = \Delta_C + \Delta_{SO} + \Delta_\delta + \frac{1}{2}(E_i - E_f)$.

Another source of the spin-isospin symmetry violation is the particle-particle interaction and, in particular, the nucleon pairing. The relative contribution of this source to the violation intensity can be estimated as the ratio $\Delta_{n,p}/\Delta_{ls}$ (Δ_{ls} is the mean energy of the spin-orbit splitting near the Fermi energy), which is rather small ($\leq 20\%$) for intermediate and heavy mass nuclei. Thus, we do not expect of the essential dependence of amplitude M_{GT} calculated within the framework of the proposed method on the particle-particle interaction strength in contrast to the standard methods (see e.g. [2]).

4. Let us turn to the choice of model parameters and to calculation results. The parametrization and phenomenological parameters of the isoscalar part of the nuclear mean field are given in detail in ref. [16]. The intensity $F' = 300\ MeV\ fm^3$ in eq. (11) is chosen to describe the experimental neutron and proton bound-energy difference for nuclei with rather large neutron excess $^{48}Ca,^{68}Ni,^{132}Sn,^{208}Pb$ [14], for which the difference is mainly determined by the symmetry potential and mean Coulomb field. The intensity $G' = 255\ MeV\ fm^3$ in eq. (11) is chosen according to ref. [17]. The parameters $\mu_{n,p}$ and $\Delta_{n,p}$, which are used for calculation of the Bogoliubov-transformation coefficients, were found for each subsystem with strong nucleon pairing so that to satisfy eqs. (14) and to describe the experimental pairing energies $P = 2\min_\lambda \mathcal{E}_\lambda$ taken according to ref. [14]. The pair-vibration state energies $2\Delta = P$ are calculated on the experimental pairing energies [14]. Coefficients c_λ determining the pair-vibration state wave function for a nucleon subsystem "magic + or – two nucleons", are calculated according to equations $c_\lambda \sim (1-2n_\lambda)/(\epsilon_\lambda - \epsilon_1 + 2\Delta)$ or $c_\lambda \sim (1-2n_\lambda)/(\epsilon_0 - \epsilon_\lambda + 2\Delta)$ with taking into account the normalization conditions given in Sect.3 [15]. For all considered nuclei the single-particle basis including all bound states as well as the quasi-bound states up to $\sim 5MeV$ is used. If the nucleon subsystem in a parent nucleus is "magic $\pm$ two nucleons", and in the product-nucleus is "magic $\pm$ four nucleons", the calculations were performed within the framework of the BCS model with the use of u_λ, v_λ factors calculated for a "magic $\pm$ four nucleons" subsystem.

Calculation results and corresponding experimental data for a wide range of nuclei are given in Table 1. The M_{GT} amplitudes calculated by eq. (16) are given (in m_e^{-1}, m_e is the electron mass) in column 5. The M^0_{GT} amplitudes were calculated by eq. (17) with the use of the same model parameters and the same number of basic states. The calculated hindrance factors h_{GT} are given in column 6. Half-lifes $T^{calc}_{1/2}$ calculated by formula $(T_{1/2})^{-1} = G_{2\nu}|M_{GT}|^2$ are given in column 8. The lepton factors $G_{2\nu}$ taken from refs. [1, 11, 12] are also given (column 3). We calculated also the $2\nu\beta^+\beta^+$ half-lifes (including the electron capture) for those nuclei, for which appearance of relevant experimental data is expected.

Two main conclusions follow from the data given in Table 1. (i) The calculated hindrance factors h_{GT} are found to be sufficiently small to justify the use of the perturbation theory with respect to the spin-orbital part of nuclear mean field. (ii) Except for the Te isotopes, the calculated amplitudes M_{GT} within the factor 2 – 3 are in agreement with corresponding amplitudes M^{exp}_{GT} deduced from experimental data.

The following step in the analysis of the $2\nu\beta\beta$ – decay amplitudes by the proposed method can be the evaluation of amplitude (9) with the use of the intermediate-state energies and wave functions calculated within the QRPA. As mentioned above, the main reason for the hindrance of the $2\nu\beta\beta$ – decay amplitude, which is caused by existence of the GTS, has been taken into account

in eq. (9). That is why we do not expect of essential change of amplitudes M_{GT} as compared with calculated ones. A consideration for the particle-hole interaction (for instance, in form (11)) will cause to some redistribution of the small GT strength over the low-lying intermediate states (AGTS) without the change of their total strength. This latter is mainly determined by the difference of the sum rule $3e_q^2(N-Z)$ and the strength exhausted by the GTS. Nevertheless, because the $2\nu\beta\beta$ – decay amplitude is formed at the expence of a rather small number of the AGTS (this statement follows from the analysis of numerical calculations of M_{GT}), the consideration for the quasiparticle interaction can result in some change of M_{GT}. The further analysis will allow to specify these reasons and, probably, to improve the description of experimental data.

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Table 1. Explanations are given in the text.

parent nucleus	type of decay	$G_{2\nu}$ $years^{-1}\ m_e^2$		$M_{GT}^{exp.}$ m_e^{-1}	M_{GT} m_e^{-1}	h_{GT}	$T_{1/2}^{exp.}$ $years$		$T_{1/2}^{calc.}$ $years$
^{76}Ge	$\beta^-\beta^-$	1.317×10^{-19}	[11]	0.0919	0.0387	0.131	0.9×10^{21}	[18]	5.0×10^{21}
				0.0737			1.43×10^{21}	[19]	
^{78}Kr	$ec\ ec$	1.957×10^{-21}	[12]		0.0285	0.090			6.2×10^{23}
	β^+ec	1.174×10^{-21}	[12]						1.0×10^{24}
^{82}Se	$\beta^-\beta^-$	4.393×10^{-18}	[11]	0.0459	0.0295	0.093	1.08×10^{20}	[20]	2.6×10^{20}
^{96}Zr	$\beta^-\beta^-$	1.953×10^{-17}		0.0362	0.0678	0.324	3.9×10^{19}		1.1×10^{19}
^{96}Ru	$ec\ ec$	6.936×10^{-21}	[12]		0.1005	0.338			1.4×10^{22}
	β^+ec	1.148×10^{-21}	[12]				$> 6.7 \times 10^{16}$	[21]	8.6×10^{22}
^{100}Mo	$\beta^-\beta^-$	9.553×10^{-18}	[11]	0.0954	0.1606	0.329	1.15×10^{19}	[22]	4.1×10^{18}
^{106}Cd	$ec\ ec$	1.573×10^{-20}	[12]		0.1947	0.319			1.7×10^{21}
	β^+ec	1.970×10^{-21}	[12]				$> 6.6 \times 10^{18}$	[23]	1.3×10^{22}
^{116}Cd	$\beta^-\beta^-$	8.000×10^{-18}	[1]	0.0754	0.0788	0.258	2.25×10^{19}	[1]	1.2×10^{19}
^{124}Xe	$ec\ ec$	5.101×10^{-20}	[12]		0.0528	0.085			7.0×10^{21}
	β^+ec	4.353×10^{-21}	[12]						8.2×10^{22}
^{128}Te	$\beta^-\beta^-$	8.624×10^{-22}	[11]	0.0123	0.0529	0.090	7.7×10^{24}	[24]	4.1×10^{23}
^{130}Te	$\beta^-\beta^-$	4.849×10^{-18}	[11]	0.0087	0.0468	0.085	2.7×10^{21}	[24]	9.4×10^{19}
^{130}Ba	$ec\ ec$	4.134×10^{-20}	[12]		0.0568	0.082	$> 4 \times 10^{21}$	[25]	7.5×10^{21}
	β^+ec	1.387×10^{-21}	[12]				$> 4 \times 10^{21}$	[25]	2.2×10^{23}
^{136}Xe	$\beta^-\beta^-$	4.870×10^{-18}	[11]	0.0299	0.0341	0.088	$> 2.3 \times 10^{20}$	[26]	1.1×10^{20}
^{136}Ce	$ec\ ec$	3.988×10^{-20}	[12]		0.0512	0.081			9.6×10^{21}
	β^+ec	6.399×10^{-22}	[12]						6.0×10^{23}
^{150}Nd	$\beta^-\beta^-$	1.200×10^{-16}	[11]	0.0221	0.0642	0.182	1.7×10^{19}	[27]	2.0×10^{18}
				0.0304			9×10^{18}	[28]	